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INTEGRAL FORM OF A HYDRAULIC EQUATION OF A STRATIFIED-DENSITY FLOW, WHEN THE LOWER FLOW IS THE COLLAPSED ROCK MASS INTRUDING INTO THE WATER RESERVOIR UNDER THE ACTION OF SEISMIC FORCES

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Abstract: *In the published fundamental works about hydrodynamic theory of mud streams, avalanches and landslides, the calculation of their characteristics, seismic forces have not been taken into consideration. That is why we additionally introduce the seismic force into the dynamic equation since the crumbling slope is potentially dangerous, because it may be set in motion under the action of a seismic shock of certain intensity and direction.*

Key words: *stratified-density flow; mud stream; avalanches; landslides; seismic shock; Coulomb friction; sloping slide surface; collapsed rock stream; phenomenological theories of macroscopic physics.*

1. INTRODUCTION

In fundamental works about hydrodynamic theory of mud streams, avalanches and landslides, seismic forces have not been taken into consideration. We additionally introduce the seismic force into the dynamic equation since the crumbling slope is potentially dangerous, because it may be set in motion under the action of a seismic shock of certain intensity and direction.

2. THE BODY OF THE ARTICLE

In the published fundamental works [1,2], an equivalent approximation of Coulomb friction for the motion of loose granulated media is in fact the expression

$$f_k = \frac{tg\varphi}{1+2tg^2\varphi} \tag{1}$$

since if the loose medium is "frozen" and a mechanical equation is written for the motion of a solid body along the sloping slide surface, then the slide surface slope, as a characteristic of the motive force in the gravitational field will be resisted by the Coulomb friction coefficient, as a value equal to the ratio of friction force to the force of normal reaction of the body weight.

We have

$$f_k = \frac{0{,}5tg2\varphi}{\sqrt{1+tg^2 2\varphi}}, \tag{2}$$

which yields large values for f_k, more closely agreeing with the tabular values of the Coulomb friction coefficient [3].

In engineering practice, seismic forces are, as known [4], taken into consideration via the dimensionless seismicity coefficient k_c (that is normalized in [4]. Its product by the weight of the structure, or structural elements, or that part of the rock massif, that is potentially apt to collapse gives the seismic force value, which is to be taken into account in deriving equilibrium equations for investigated bodies, media and the like.

Horizontal seismic forces are usually regarded, as most dangerous in the sense of the loss of the stability of structures, sloping surfaces and so on.

If, for instance, we consider the resistance of some body to the slide along the surface with a slope angle ψ with respect to the horizon under the action of gravitational and seismic forces (Fig. 1), then a limiting equilibrium equation will be written in the form

$$T + p_c \cos\psi - T' \leq 0, \tag{3}$$

where the principal shearing force preconditioned by the slope is

$$T = G \sin\psi = \rho g w \sin\varphi; \tag{4}$$

is the density; w is the body volume; g =9.81 m/s;

the seismic force

$$p_c = k_c G = k_c \rho g v \tag{5}$$

and the principal force resisting to the slide

$$T' = f\left[N - p_c \cos\left(\frac{\pi}{2} - \psi\right)\right] = f\left(Np_c \sin\psi\right), \tag{6}$$

where N is the normal reaction (N=Gcosψ= gvcosψ) weakened due to the seismic force component in the direction of the normal n-n.

According to the stability condition (35), the body in the state of rest may lose its stability, if the following condition is fulfilled

$$gv(\sin\psi + k_c\cos\psi - f\cos\psi + fk_c\sin\psi) > 0\ , \tag{7}$$

i.e. for

$$\sin\psi\ (1+k_c f) > \cos\psi\ (f-k_c)\ , \tag{8}$$

or

$$tg\psi > \frac{f-k_c}{1+k_c f}\ . \tag{9}$$

By (8), the seismicity coefficient, that accounts for the body stability loss on the sloping surface is

$$k_c > \frac{f-tg\psi}{1+ftg\psi}\ . \tag{10}$$

Thus, if f>tgψ, i.e. the body cannot slide on the sloping plane, because it is in the stable state of rest, then for the body not to lose its stability and start to slide downward the seismicity coefficient value must exceed the right-hand part of (10).

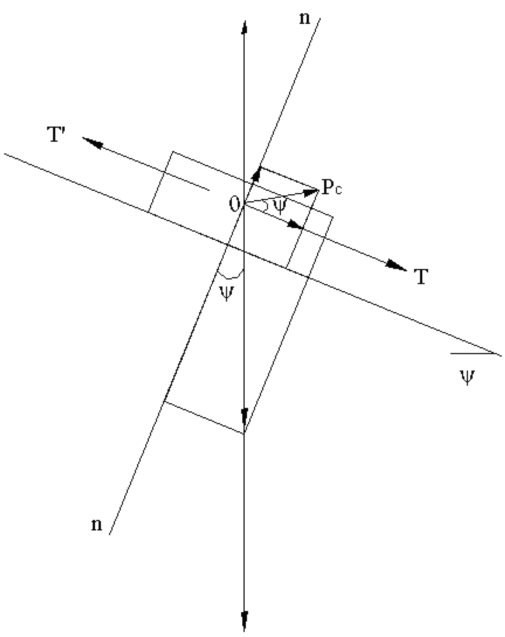

Fig. 1

Condition (7) shows how to take into account the seismic action on the rock motion condition. If we assume, that the Coulomb friction coefficient is the complex value

$$f_k \cong \frac{f}{1+2f^2},$$

or

$$f_k = \frac{0{,}5tg2\varphi}{\sqrt{1+tg^2 2\varphi}},$$

then it is obvious, that the condition of static stability of the rock, with non-plastic property can be written as

$$i_0 = \sin\psi < f\cos\psi, \quad (11)$$

which = for s=1 and is equivalent to the condition

$$\sin\psi < f_k\cos\psi . \quad (12)$$

The latter expression coincides with (8) for k_c =0 (the absence of seism).

When the rock stream is subjected to the action of seismic forces, by virtue of (40) the static equilibrium condition will be written as an inequality

$$\sin\psi\,(1+k_c f_k) < \cos\psi\,(f_k - k_c). \quad (13)$$

This makes clear the technique, by which the rock motion takes the seismic effect into account. In accordance with the above, the multipliers $(1+k_c f_k)$ and $(f_k - k_c)$ are respectively introduced into the slope value $i_0 = \sin\psi$ and the Coulomb friction value.

Hence, a generalized equation of the dynamics of a rock stream moving along the underwater slope under the action of a horizontal seismic force has the form

$$\times\cos\psi h\frac{\partial h}{\partial x} + h\frac{\partial p_w}{\partial x} - \gamma h\left[\sin\psi(1+k_c f_k) - (f_k - k_c)\cos\psi\right] + \tau_0 +$$

$$+\rho_w k_f \frac{(v-w)^2}{2} + k\bar{s}\left[\cos^2\varphi - \frac{\sin^2 2\varphi}{4(1+\sin^2\varphi)}\right] = 0. \quad (14)$$

Written in this form, the equation is applicable for a composite loose medium, whose pores are filled with water containing clayey inclusions and which thus has (in addition to the cohesion k) plastic properties, defined by the threshold shearing stress $_0$.

In the sequel, the plastic properties (k = 0 and $_0$ =0) will be neglected, as because of their smallness they do no play any role for the rock stream consisting mainly of broken stone pieces.

Therefore for the intrusion of the rock stream into Lake Sarez or any other water reservoir, equation (14) reduces to

$$\frac{\partial}{\partial t}\left(\alpha_t \rho_t wh\right) + \frac{\partial}{\partial x}\left(\alpha_0 \bar{\rho} w^2 h\right) + \beta h \frac{\partial h}{\partial x} + h\frac{\partial p_w}{\partial x} -$$

$$-\gamma I_s h + \rho_w k_f \frac{(v-w)^2}{2} = 0, \qquad (15)$$

where there is the multiplier of $h\frac{\partial h}{\partial x}$ in (46), and I_s is the effective slope,

$$I_s = \sin\psi\ (1+k_c f_k) - (f_k - k_c)\cos\psi\ ,$$

here s=1 since the water content in rock pores can be assumed to be negligibly small (in the rock and water mixture volume).

Following the procedure, that T.G. Voinich-Syanozhentski applied to the dam breach problem, when considering the dynamics of mudflows and avalanche-like streams [1], we integrate equation (15) with respect to the longitudinal co-ordinate x for the whole collapsed rock stream body, keeping in mind, that during the downward motion the rock stream length changes insignificantly, as different from the flow of a visco-plastic fluid, for which the slide condition gives an increase in the longitudinal dimension of its wave body.

We obtain

$$\frac{\partial}{\partial t}\int_{\xi_1}^{\xi_2} \alpha_t \bar{\rho} whdx - \left(\alpha_t \bar{\rho} wh\right)_{\xi_2} \frac{\partial \xi_2}{\partial x} + \left(\alpha_t \bar{\rho} wh\right)_{\xi_1} \frac{\partial \xi_1}{\partial t} +$$

$$+\left(\alpha_0 \bar{\rho} w^2 h\right)_{\xi_2} - \left(\alpha_0 \bar{\rho} w^2 h\right)_{\xi_1} + \beta_1 \left(\frac{h^2}{2}\right)_{\xi_2} - \beta_1 \left(\frac{h^2}{2}\right)_{\xi_1} +$$

$$+\bar{h}\left(p_{w_2} - p_{w_1}\right) - \left(\gamma - \gamma_h\right)\bar{I}_s v + \rho_w \frac{k_f}{2}\int_{\xi_1}^{\xi_2} w^2 dx = 0, \qquad (16)$$

where $v = \int_{\xi_1}^{\xi_2} hdx$ is the wave body volume of the collapsed rock stream).

Since the height of the collapsed rock stream body is zero at the beginning and at the end of its motion, $h_{\xi_2} = h_{\xi_1} = 0$ and thus vanish in the second to the sixth term inclusive. Then, (16) is written in the simplified form (neglecting the last summand)

$$\frac{d(\tilde{c} v \rho)}{dt} = \gamma \bar{I}_s v - h\left(p_{w_2} - p_{w_1}\right), \qquad (17)$$

where $\widetilde{c}$ is the velocity of the center of the wave body mass volume of the collapsed rock stream intruding into the water reservoir and subjected to the action of the external water medium.

Since, during the intrusion of the wave body of the collapsed rock stream into the water reservoir its lower part moves along the underwater slope and the upper part moves along the dry slope and the coefficient of Coulomb friction against the wetted surface f_{kw} is always smaller, than that in the absence of wettedness $f_{ks}(f_{ks} > f_{kw})$, we must write the separate motion equation for the upper part of the collapsed rock, which moves along the dry slope. It is obvious, that it will be differ from equation (17), only in the absence of the last summand.

Thus, we have two equations

$$\frac{d(\widetilde{c}_s v_s \bar{\rho}_s)}{dt} = \gamma_s \bar{I}_s v_s + R_{sw} \tag{18}$$

and

$$\frac{d(\widetilde{c}_w v_w \bar{\rho})}{dt} = (\gamma - \gamma_w)\bar{I}_w v_w - R_{ws} - \bar{h}(p_{w_2} - p_{w_1})^1 , \tag{19}$$

where v_s and v_w are the volumes of the surface and underwater parts of the total volume v_0 of the collapsed rock body, $\widetilde{c}_s$ and $\widetilde{\mathbf{c}}_w$ are the mass center velocities corresponding to v_s and v_w, $R_{sw} = R_{ws}$ are the interaction forces between these two parts of the collapsed rock slide body, $\bar{I}_s$ and $\bar{I}_w$ are the "effective" slopes corresponding to the dry and the underwater part of the slide surface.

Let us assume approximately that

$$v_w = v_0 \frac{l_w}{L_s}, \tag{20}$$

where L_s is the full length of the collapsed rock wave body; l_w is the longitudinal extension of its underwater part.

Let v_s be also the volume of that collapsed-rock slide, which at the current moment of time is above the reservoir water level, so that $v_0 = v_s + v_w$ (v_0 is the total volume of the collapsed rock stream)

[1] This form of (19) actually means, that from $\frac{\partial p_w}{\partial x}$ we have eliminated- the hydrostatic component (contained in the first summand of the right-hand part), while p_{w_1} and p_{w_2} are the dynamic components giving respectively the head resistance and the adjoint mass.

If p_{w_1} corresponds to the water-edge and p_{w_2} to the front of the wave body of the collapsed rock slide, that has intruded into the water reservoir, then $p_{w_1}=0$, while p_{w_2} is defined (with regard for the resistance, caused in particular by the non-stationary state of the process) by the well-known hydro-aero-dynamic expression for head resistance

$$p_{w_2}=k_w\rho_w\frac{\overline{c}^2}{2}+\beta\rho_w\frac{d(\widetilde{c}v)}{dt}, \tag{21}$$

where $\beta=\dfrac{v^*}{v_w}$ is the coefficient of the adjoint mass v*, while $\widetilde{\mathbf{c}}$ is the reduced average velocity of the transport motion of the collapsed rock slide body, with total mass v_0.

Summing (18) and (19) with (20) and (21) taken into account, we obtain

$$\begin{aligned}&\frac{d}{dt}\left(\rho_s\overline{c}_s v_s+\overline{\rho c}_w v_w\right)\cong\frac{d}{dt}\left(\overline{\rho}_s\widetilde{c}v_0\right)=\\&=\gamma_s\bar{I}_s\left(v_0-v_w\right)+\left(\gamma-\gamma_w\right)\bar{I}_w v_0\frac{l_w}{L_s}-\\&-k_w\rho_w\frac{\widetilde{c}^2}{2}\bar{h}\rho_w\beta-\frac{d}{dt}\left(\widetilde{c}v_0\frac{l_w}{L_s}\right),\end{aligned} \tag{22}$$

where

$$\widetilde{c}=\frac{l_s c_s v_s+\rho_w c_w v_w}{\overline{\rho}_s v_0}. \tag{23}$$

The equation (22) can be written in a more compact form:

$$\begin{aligned}&\left(1+\beta\frac{l_w}{L_s}\frac{\rho_w}{\rho_s}\right)\frac{d\widetilde{c}}{dt}=gI_s-g\left(I_s-\sigma_s I_w\right)\frac{l_w}{L_s}-\\&-k_w\rho_w\frac{h_{sm}}{h_s}\frac{l_{sm}}{L_s}\frac{\widetilde{c}^2}{2h_s},\end{aligned} \tag{24}$$

where $\sigma_s=1-\dfrac{\rho_w}{\rho_s}$.

In [3], an analogous equation for the wave body of the collapsed rock stream, that has intruded into Lake Sarez was obtained using semi-heuristic argumentation and somewhat obscure formulations, as to its macroscopic structure.

An equation derived in (24) has the form

$$\left(1+\beta\frac{\rho_w}{\rho_s}\frac{l_w}{l_s}\right)L_s\frac{dv_s^2}{dL_{sw}}=2gI_sL_s-2gL_s\left(I_s-G_sI_w\right)\frac{l_w}{L_s}-$$

$$-\frac{\rho_w}{\rho_s}\left(\beta k_w \frac{l_{sm}}{L_s}\right)v_s^2, \qquad (25)$$

where $\mathbf{v}_s \cong \tilde{\mathbf{c}}$ (in our notation), while all other symbols coincide completely.

Since $v_c = \tilde{c} = \frac{dl_w}{dt}$, we easily observe, that (24) and (25) are practically identical [1]:

$$I_s = \sin\psi_s(1+k_c f_{ks}) - (f_{ks} - k_c)\cos\psi_s; \qquad (26)$$

$$I_w = \sin\psi_w(1+k_c f_{kw}) - (f_{kw} - k_c)\cos\psi_w, \qquad (27)$$

where f_{ks} and f_{kw} are the coefficients of Coulomb friction against the under- and above-water slopes.

In [5], the equation (24) is used, as a design equation due to the initial postulation of a sudden occurrence of constant seismic load within 30 seconds, i.e. it was assumed, that $k_c > 0$ and it is equal to the constant value 0.1 during this time, upon the lapse of which $k_c = 0$

This precondition made it possible to estimate a maximal possible distance (range) of the intrusion of the collapsed rock stream into Lake Sarez and, with its aid, a largest possible height of the wave generated in the lake by this intrusion.

However in reality the seismicity coefficient is usually a value close to the periodic attenuation function. In order to take this fact into consideration it is necessary to use the dynamic equation of a landslide intruding into a water reservoir in form (24).

3. CONCLUSION

Thus, the procedure of derivation of equation (24) based on a dynamic equation of the mechanics of a composite loose medium with plastic properties, i.e. on general and strict equations of the mechanics of analogous continuous media confirms, that the equation derived in [3] by semi-heuristic arguments has a sufficiently reliable theoretical justification (in the framework of phenomenological theories of macroscopic physics).

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[1] In the equation (24) (in our numeration) which was resistance of the upper surface of the landslide body and which we neglect in deriving (24) because of its smallness.

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