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Engineering

Numerical Computation of Wave Motions for Poti Coastal Zone

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(Presented by Academy Member Archil Prangishvili)

In Poti and its littoral the ecological and geomorphological problems exist over several ten years. These issues are related to concstruction of hydro engineering buildings, which were designed and realized unsuccessfully for various purposes. The problems related to geomorphological nature of Poti coastal strip began in 1939 when river Rioni was entirely thrown-over to the northward of city. Although, this action saved the city from frequent floods but at the boundary, because the deficit of beach an isle was shaped. The sea washed away the Poti coastal strip, which was retreated up to several hundreds of meters. This paper develops a mathematical model of the wave movements of the Poti coastline. The basic equations of water discharge were solved numerically applying method of finite elements and Crank-Nicholson scheme. According to the model, computational experiments were carried out. The changes in the free surface elevation, transversal and longitudinal sea currents were computed. While testing the model, calculations of the wave mode in deep water under the same boundary conditions were performed in this region. The results of the calculations coincide with the data known analytical solutions. *© 2020 Bull. Georg. Natl. Acad. Sci.*

Wave motion, Geomorphology, coastal erosion, sediments, numerical model.

Locations of geomorphological and hydro-engineering problems existing in Poti sea region are schematically given in Fig.1.

The grave environmental problems began in Poti after transfer of the main flow of the river Rioni to the north. As a result, the flooding of the city stopped, however the reduction of water consumption in the city channel, caused a decrease of sediments carried away by the river causing a coastal erosion (Fig. 1).

The coast changes are referred the movement of the waves and currents in the coastal part of the sea. We consider the wave movements in the coastal zone of the sea in the Poti region.

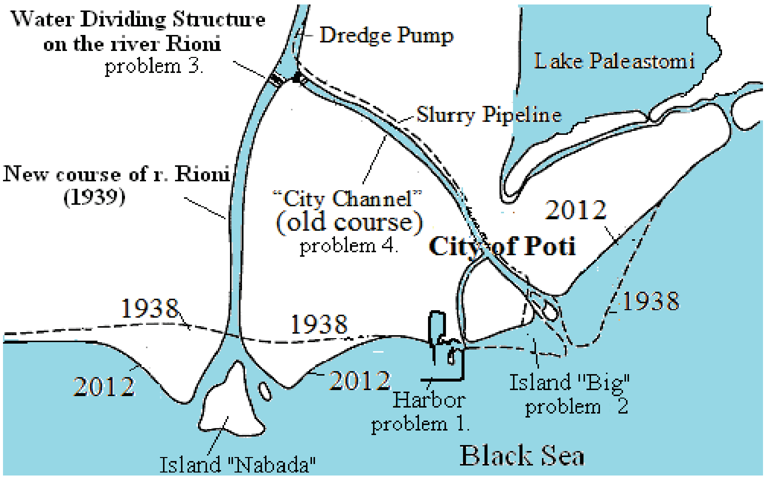

Fig. 1. Schematic map of Poti sea region.

--- coastal shore line in 1938; — coastal line in 2012.

Numerical Model of Wave Motion of the Sea

The basic equation of wave motion is [1-3]:

$$\begin{aligned} &\frac{\partial \vec{Q}}{\partial t}+\frac{c^2}{n}\vec{\nabla}(n\varsigma)+f\vec{Q}=0, \\ &\frac{\partial \varsigma}{\partial t}+\vec{\nabla}\cdot\vec{Q}=0. \end{aligned} \tag{1}$$

where $\vec{Q}$ is the vector fluid flow $\vec{Q}=\int_{-h}^{\zeta}\vec{u}dz$; $\zeta(t,x_1,x_2)$ – the free surface of the calm sea level elevation; n – the group velocity; c_g related to the phase velocity c ; f – coefficient of bottom friction; h – water depth. The following relations hold [2,3]: $\sigma^2=gk\cdot\tanh(kh)$, $c=\frac{\sigma}{k}=\sqrt{\frac{g}{k}\tanh(kh)}$, $n=\frac{c_g}{c}=\frac{1}{2}\left(1+\frac{2kh}{\sinh(2kh)}\right)$, where $\sigma=\frac{2\pi}{T}$ is the angular frequency; $k=\frac{2\pi}{L}$ – wavenumber; T, L – respectively, the period and wavelength.

According to the above mentioned, we use the following equations :

$$\begin{aligned} &\frac{\partial \vec{Q}}{\partial t}+\frac{c^2}{n}\vec{\nabla}\xi=0 \\ &\frac{\partial \xi}{\partial t}+n\vec{\nabla}\cdot\vec{Q}=0 \end{aligned} \tag{2}$$

where the function $\xi(t,x_1,x_2)=n(x_1,x_2)\cdot\zeta(t,x_1,x_2)$ adjusted increase of sea free surface.

We would like to explain the boundary conditions. On the part B_1 of the common boundary B of the integration domain, a vector function – $\vec{Q}^*(t,x_1,x_2)$ is given, on the part B_2 is a function $\xi^*(t,x_1,x_2)$.

The boundary value problem for equation (2) with the corresponding boundary conditions is equivalent to the variation problem of the form:

$$\begin{aligned}
&\int_S \left(\frac{\partial \vec{Q}}{\partial t} + \frac{c^2}{n}\vec{\nabla}\xi\right)\cdot \delta\vec{Q}dS = \int_{B_2} \frac{c^2}{n}(\xi-\xi^*)\vec{N}\cdot\delta\vec{Q}dB,\\
&\int_S \left(\frac{\partial \xi}{\partial t} + n\vec{\nabla}\cdot\vec{Q}\right)\delta\xi dS = \int_{B_1} n(\vec{Q}-\vec{Q}^*)\cdot\vec{N}\delta\xi dB,
\end{aligned} \tag{3}$$

where $\delta\vec{Q}$ and $\partial\xi$ are variations meet the boundary conditions for the flow and elevation, $\vec{N}$ is the unit vector of the outer normal to the boundary of the integration region. Integration in the left-hand sides of the equations is carried out over the entire region of *S*, in the right-hand sides by the corresponding parts of the boundary.

From the Eq.3 we obtain:

$$\begin{aligned}
&\int_S \left(\frac{\partial \vec{Q}}{\partial t}\cdot\delta\vec{Q} - \xi\vec{\nabla}\cdot\left(\frac{c^2}{n}\delta\vec{Q}\right)\right)dS + \int_{B_2}\frac{c^2}{n}\xi^*\vec{N}\cdot\delta\vec{Q}dB = 0\\
&\int_S \left(\frac{\partial \xi}{\partial t}\delta\xi - \vec{Q}\cdot\vec{\nabla}(n\delta\xi\right)dS + \int_{B_1} n\vec{Q}^*\cdot\vec{N}\delta\xi dB = 0
\end{aligned} \tag{4}$$

We divide the domain of integration into finite elements of $S=\bigcup S_e$. On each finite element S_e we use the following function approximations [4]:

$$Q_i(t,x_1,x_2) = Q_i^N(t)\psi(x_1,x_2);\quad \delta Q_i(t,x_1,x_2) = \delta Q_i^N(t)\cdot\psi(x_1,x_2);$$

$$\left(\frac{c^2}{n}\right) = \left(\frac{c^2}{n}\right)^N \psi(x_1,x_2);\ n(x_1,x_2) = n^N\psi(x_1,x_2)\ \ldots$$

where $\psi(x_1,x_2)$ are local interpolation (basis) functions.

When solving the Cauchy problem for the system Eq. 4, one can use the method of finite differences. On the time axis t we take the discrete set of points $t_0, t_1, t_2, ..., t_{i-1}, t_i, ...$ with step Δt and use the Crank-Nicolson scheme [5].

On the basis of (4) we obtain a system of linear algebraic equations:

$$\begin{aligned}
&\sum_{(e)}\overset{(e)}{\Omega}{}^K_\Delta\left[\frac{a_{NK}}{\Delta t}Q^N_{1(i)} + b_{NMK1}\left(\frac{C^2}{n}\right)^M\frac{\xi^N_{(i)}}{2}\right] = \sum_{(e)}\Omega^K_\Delta\left[\frac{a_{NK}}{\Delta t}Q^N_{1(i-1)} - b_{NMK1}\left(\frac{c^2}{n}\right)^M\frac{\overset{(e)}{\xi}{}^N_{(i-1)}}{2} + \left(\frac{c^2}{n}\right)^M\frac{\xi^{*N}_{(i)}+\xi^{*N}_{(i-1)}}{2}p_{NMK1}\right],\\
&\sum_{(e)}\overset{(e)}{\Omega}{}^K_\Delta\left[\frac{a_{NK}}{\Delta t}Q^N_{2(i)} + b_{NMK2}\left(\frac{C^2}{n}\right)^M\frac{\xi^N_{(i)}}{2}\right] = \sum_{(e)}\Omega^K_\Delta\left[\frac{a_{NK}}{\Delta t}Q^N_{2(i-1)} - b_{NMK2}\left(\frac{c^2}{n}\right)^M\frac{\overset{(e)}{\xi}{}^N_{(i-1)}}{2} + \left(\frac{c^2}{n}\right)^M\frac{\xi^{*N}_{(i)}+\xi^{*N}_{(i-1)}}{2}p_{NMK2}\right]\Rightarrow\\
&\Rightarrow\sum_{(e)}\overset{(e)}{\Omega}{}^K_\Delta\left[\frac{a_{NK}}{\Delta t}\xi^N_{(i)} + b_{NMK1}n^M\frac{Q^N_{1(i)}}{2} + b_{NMK2}n^M\frac{Q^N_{2(i)}}{2}\right] =\\
&=\sum_{(e)}\overset{(e)}{\Omega}{}^K_\Delta\left[\frac{a_{NK}}{\Delta t}\xi^N_{(i-1)} - b_{NMK1}n^M\frac{Q^N_{1(i-1)}}{2} - b_{NMK2}n^M\frac{Q^N_{2(i-1)}}{2} + n^M\left(\frac{Q^N_{1(i)}+Q^N_{1(i-1)}}{2}N_1p_{NMK1} + \frac{Q^N_{2(i)}+Q^N_{2(i-1)}}{2}N_2p_{NMK2}\right)\right].
\end{aligned} \tag{5}$$

At time t_0, the values of nodal flows $Q^N_{1(0)}$, $Q^N_{2(0)}$ and the elevations of the free surface $\xi^N_{(0)}$ are give the initial conditions. To find the wave field at the i-the time step, we must solve the system of equations (5) with respect to the unknowns $Q^N_{1(i)}$, $Q^N_{2(i)}$, $\xi^N_{(i)}$, taking into account the boundary conditions.

For studying the coastal wave mode in the area of Poti, we consider a section of the sea in the plan of 700×600 m, adjacent to the Poti port.

The domain of integration was divided into triangular finite elements, the steps of the partition were taken as follows: along the x_1 axis $\Delta x_1 = 5$ m, along the x_2 axis – $\Delta x_2 = 50$ m. Number of nodes along the first x_1 axis equals 141, $i = 1 \div 141$, along the second x_2 axis equals 50 $(j = 1 \div 13)$.

The parameters of waves in deep water are as follows: the direction of propagation is the west (along the x_1); wave height is $H = 1$ m; wave period is T=4s.

When performing calculations in the equations of motion of a mathematical model, an additional term corresponding to the energy dissipation due to wave embedding, when approaching the shore, is introduced [1]. Therefore, the amplitudes of the oscillations, of the elevation of the free surface immediately near the shore decrease. For the purpose of testing the model, algorithm and program, computation was made of the wave mode in deep water in the region considering of the same boundary conditions. The results of computation utterly correspond to admitted analytical solutions.

The results of the numerical solution of the model are shown in Figures 2-4.

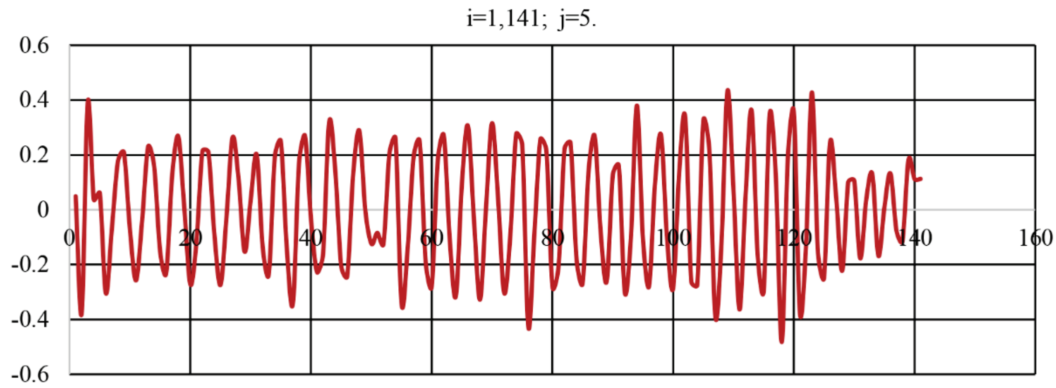

Fig. 2. Changes in the elevation of the free surface of the sea near the shore (i=70-141).

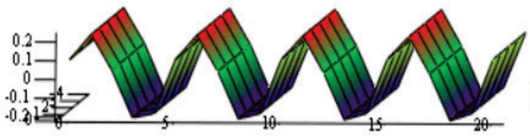

Fig. 3. Fragment of the wave pattern in deep water.

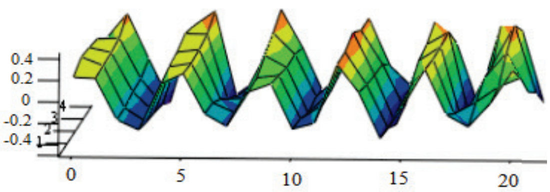

Fig. 4. Fragment of the wave pattern with the existing bottom profile.

In Fig. 2 shows changes in the height of the free surface of the sea ξ. The changes of the transverse coast is q_1 and the coastal flows is q_2 of velocity in the following areas are calculated $(i = 70 \div 141,\ j = 1 \div 4)$ for the moments of time corresponding to phases equals 180° and 225° of one wave period. In Fig. 4, the scale lengths along the vertical and horizontal lines are different, the length of the section for the range is 350 m.

Conclusions

A mathematical model of wave motions of the sea in the coastal zone of the Poti region of the Black Sea has been developed. To solve the basic equations of the model, the finite element method is used. When solving the Cauchy problem, we use the finite difference method and the Crank-Nicolson scheme. A program was developed for the numerical solution of the equations obtained. As a result, changes in the height of the free sea surface ξ, transverse flow q_1 and alongshore flow q_2 were obtained. For the purpose of testing the model, algorithm and program, computation was made of the wave mode in deep water in the region under consideration of the same boundary conditions.

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ინჟინერია

ტალღურ მოძრაობათა რიცხვითი გამოთვლები ფოთის სანაპირო ზოლისთვის

ი. საღინაძე*, მ. კოდუა**, მ. ფხაკაძე*

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ქალაქ ფოთსა და მის საზღვაო რეგიონში რამდენიმე ათეული წელია არსებობს გარემოსდაცვითი და გეომორფოლოგიური ხასიათის დღემდე გადაუჭრელი პრობლემები, რაც განაპირობა ამ რეგიონში, სხვადასხვა წლებში და სხვადასხვა მიზნით წარუმატებლად დაპროექტებულმა და განხორციელებულმა ჰიდროსაინჟინრო მშენებლობებმა. გეომორფოლოგიური ხასიათის ეს პრობლემები ქალაქ ფოთის სანაპირო ზოლში დაიწყო მას შემდეგ, რაც 1939 წელს მდ. რიონი მთლიანად იქნა გადაგდებული ქალაქის ჩრდილოეთით. ამ ღონისძიებამ, მართალია, თვით ქალაქი იხსნა ხშირი დატბორვებისგან, მაგრამ ქალაქის ნაპირზე შექმნა პლაჟწარმომქმნელი ნატანის აუნაზღაურებელი დეფიციტი. ზღვამ კატასტროფულად წარეცხა სანაპირო ზოლი და ასეულობით მეტრით დაახევინა უკან. ეროზიული პროცესები გამოწვეულია ტალღური მოძრაობებითა და სანაპირო დინებებით. ნაშრომში შემოთავაზებულია ქალაქ ფოთის შავი ზღვის სანაპირო ზოლში ტალღური რეჟიმების ანგარიში სასრულ ელემენტთა მეთოდისა და კრანკლ-ნიკოლსონის სქემის გამოყენებით. ჩატარებულია რიცხვითი ექსპერიმენტები. დადგინდა ზღვის თავისუფალი ზედაპირის აწევისა, განივი და ნაპირგასწვრივი ნაკადების სიჩქარეთა ცვლილება. ტალღური მოდელის ტესტირებისათვის ჩატარებულია ტალღური რეჟიმების ანგარიში იმავე სასაზღვრე პირობებში ღრმა წყლისათვის.

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